Markov Chains: Mixing Times and Applications (IMT 3800)

First semester, 2022

Instructor:
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Class web page:
https://cursos.canvas.uc.cl/courses/43465

Class meeting times:
Tue & Thu 11:30 am – 12:50 pm
Room H4

Office hours:
Tuesday 2:00 pm – 3:20 pm
(or by appointment)

Please note: Announcements and other information about the class will be posted regularly on the Canvas web page.

Course description:

The theory of Markov chains has a wide range of applications in computer science, statistical physics, engineering, data science, and many other areas. For instance, when investigating the complexity of Markov Chain Monte Carlo (MCMC) algorithms in statistics, one is naturally led to the problem of estimating the time it takes for a Markov process to reach stationarity. Our main focus this semester will be on developing the theoretical machinery that enables us to compute such "mixing times" as a function of the size and geometry of the underlying state space. Topics covered will include the basic theory of finite Markov chains, the total variation distance, coupling estimates of mixing times, ergodicity, random walks on networks, spectral representations, Dirichlet forms, the transportation metric, and path coupling, among others. Along the way, we will also discuss and analyze various MCMC approaches to sampling and optimization, as well as certain interacting particle systems, which are of interest in computer science and statistical physics.

Learning outcomes:

Upon successful completion of this course, students will be able to demonstrate the following competencies:

- An understanding of the modern theory of finite Markov chains.
- A thorough understanding of the complexity of MCMC sampling and optimization algorithms in data science and statistics.
- An ability to engage with research literature.

References:

Our main reference will be:

- David Levin and Yuval Peres. **Markov Chains and Mixing Times**. 2nd Edition, American Mathematical Society, 2017.

There is also an excellent set of notes by Aldous and Fill that covers a lot of the material we will be discussing throughout the semester:

- David Aldous and James Allen Fill, **Reversible Markov Chains and Random Walks on Graphs**. Available at: https://www.stat.berkeley.edu/~aldous/RWG/book.html

Grading policy:

The final grade will be based on homework assignments and a final take-home exam.

Homework assignments	**60%**
Final exam	**40%**

Homework assignments:

Homework problems will be handed out on a regular basis. Discussion of homework assignments with other students is encouraged, but what is handed in should be your own work. The instructor will be available during office hours to address questions and further discuss the course material and assignments with the students.

Course content:

- Introduction to finite Markov chains (irreducibility, aperiodicity, reversibility, stationary distributions)
- Metropolis and Glauber chains
- Markov chain mixing (total variation distance, convergence, ergodicity)
- Coupling estimates and mixing times
- Strong stationary times
- Random walks on networks
- Hitting times and cover times
- Spectral representations
- Dirichlet forms
- Optimal transport and path couplings
- The Propp-Wilson algorithm